R SQUARED — ESAT PREPARATION
MOCK PAPER 1 – MATHEMATICS MODULE 2

Time: 40 minutes • 27 questions • 1 mark each • no penalty for incorrect answers

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1. Curve C has equation $y = 20 - x^2$. Line L has equation $y = 4$. What is the area enclosed between C and L?

A $\frac{196}{3}$

B $\frac{226}{3}$

C $\frac{256}{3}$

D $\frac{286}{3}$

E $\frac{316}{3}$

2. How many solutions of the equation $2\cos 3\theta = \cos\theta$ lie in the interval $-\pi \leq \theta \leq \frac{\pi}{2}$?

A 2

B 3

C 4

D 5

E 6

F 7

3. The line $y = x + k$, where k is a constant, is a tangent to the curve $y = 4x^2 - 3x + 2$. What is the value of k?

A -2

B -1

C $\frac{1}{4}$

D $\frac{1}{3}$

E $\frac{1}{2}$

F $\frac{3}{4}$

G 1

H 2

4. In a particular arithmetic progression: the 12th term is five times the 1st term; the 9th term is 3 less than twice the 4th term. What is the 3rd term of the progression?

A 7

B 11

C 15

D 19

E 23

F 27

5. Evaluate $\log_3\left(\frac{4}{3}\right) + \log_3\left(\frac{5}{4}\right) + \log_3\left(\frac{6}{5}\right) + \cdots + \log_3\left(\frac{81}{80}\right)$.

A -3

B 2

C 3

D 4

E $\log_3(4!)$

F $\log_3 80$

6. Circle C has equation $(x-2)^2+(y+5)^2=3$. The length of the tangent from the circle C to the point P is $6\sqrt{2}$. What is the shortest distance from P to C?

A $6\sqrt{2}$

B $6\sqrt{2}+\sqrt{3}$

C $4\sqrt{3}$

D $\sqrt{3}$

E 12

7. What is the coefficient of x^3 in the expansion of $(1-3x)^5(1+3x)^5$?

A -8100

B -810

C -90

D 0

E 90

F 810

G 1215

8. Given that $\displaystyle\int_0^3 (4x+m)\,dx = 15$, what is the value of m?

A $-\dfrac{5}{2}$

B -2

C $-\dfrac{3}{2}$

D -1

E $-\dfrac{1}{2}$

F 0

G 1

9. The dimensions of a solid cuboid, in cm, are x, $3x$ and y. The volume of the cuboid is 972 cm^3. At this volume, the surface area of the cuboid has its optimum value. What is the area, in cm^2, of the face that has the largest area?

A $2(486)^{2/3}$

B 54

C 108

D 162

E 486

F $4(243)^{2/3}$

10. When $x = 1$ is substituted in the expression $x^3 + px^2 + qx + p^2$, the result is 0. When $x = 3$ is substituted into the same expression, the result is 16. Find all possible value(s) of p.

A $p = -1 \pm \dfrac{\sqrt{6}}{3}$

B $p = 4$ or $p = -1$

C $p = 4$

D $p = 1 \pm \sqrt{7}$

E there are no values for p

11. A square $PQRS$ is drawn above the x-axis with side PQ on the x-axis. P is the point $(-5, 0)$ and Q is the point $(1, 0)$. A circle is drawn inside the square with diameter equal in length to the side of the square. Which one of the following is an equation of the circle?

A $x^2 + y^2 - 4x - 6y + 4 = 0$

B $x^2 + y^2 + 4x + 6y + 4 = 0$

C $x^2 + y^2 + 4x - 6y + 9 = 0$

D $x^2 + y^2 + 4x - 6y + 4 = 0$

E $x^2 + y^2 - 4x + 6y + 9 = 0$

F $x^2 + y^2 - 4x - 6y + 9 = 0$

G $x^2 + y^2 + 4x + 6y + 9 = 0$

H $x^2 + y^2 - 4x + 6y + 4 = 0$

12. The first term of a convergent geometric series is 12. The fifth term is 3. The sixth term is real and positive. What is the sum to infinity of this series? (The sum to infinity of a convergent geometric series is $\frac{a}{1-r}$, where a is the first term and r is the common ratio.)

A $12(1+\sqrt{2})$

B $12(1-\sqrt{2})$

C $12(2+\sqrt{2})$

D $12(2-\sqrt{2})$

E 24

F $\frac{12 \cdot 4^{1/5}}{4 \cdot 4^{1/5} - 1}$

G $\frac{12 \cdot 4^{1/5}}{4 \cdot 4^{1/5} + 1}$

13. Tangents are drawn from a point P to a circle of radius 6 cm. The centre of the circle is C and the distance PC is 12 cm. Which one of the following is an expression for the shaded area in square centimetres?

A $\frac{36}{3}(3\sqrt{3}-\pi)$

B $\frac{36}{3}(3\sqrt{3}+\pi)$

C $\frac{18}{3}(3\sqrt{3}-\pi)$

D $\frac{18}{3}(3\sqrt{3}+\pi)$

E $\frac{36}{3}(2\sqrt{3}-\pi)$

F $\frac{36}{3}(2\pi-3\sqrt{3})$

14. Given that $9\cos\theta - 3\tan\theta\sin\theta = 5$, which one of the following is true?

A $\cos\theta = -\frac{3}{4}$ or $-\frac{1}{3}$

B $\cos\theta = -\frac{3}{4}$ or $\frac{1}{3}$

C $\cos\theta = \frac{3}{4}$ or $\frac{1}{3}$

D $\cos\theta = \frac{3}{4}$ or $-\frac{1}{3}$

15. The complete set of values of a for which the equation $2x^2 = (a+1)x - 2$ has two real distinct roots is

A no values of a

B $-4\sqrt{2} < a < 4\sqrt{2}$

C $a < -4\sqrt{2},\ a > 4\sqrt{2}$

D $-5 < a < 3$

E $a < -5,\ a > 3$

F $-3 < a < 5$

G $a < -3,\ a > 5$

H all values of a

16. The straight line with equation $y = mx + 4$, where $m > 0$, $m \neq 1$, is perpendicular to the line with equation $y = px + 3$. The lines cut the x-axis at the points L and M respectively. The length of LM is 7 units. What is the value of $m + p$ given that $m > 1$?

A $-\frac{19}{12}$

B $-\frac{7}{12}$

C $-\frac{5}{12}$

D $\frac{5}{12}$

E $\frac{7}{12}$

F $\frac{19}{12}$

17. $f(x) = 2x^3 - a^2x$, where a is a positive constant. Find the complete set of values of x for which $f(x)$ is an increasing function.

A $x \leq -a,\ x \geq a$

B $-a \leq x \leq a$

C $x \leq -a,\ 0 \leq x \leq a$

D $-a \leq x \leq 0,\ x \geq a$

E $x \leq -\frac{a}{6},\ x \geq \frac{a}{6}$

F $-\frac{a}{6} \leq x \leq \frac{a}{6}$

G $x \leq -\frac{a}{\sqrt{6}},\ x \geq \frac{a}{\sqrt{6}}$

H $-\frac{a}{\sqrt{6}} \leq x \leq \frac{a}{\sqrt{6}}$

18. The curve $y = x^2$ is translated by the vector $\begin{pmatrix} 5 \\ 2 \end{pmatrix}$ and then reflected in the line $y = -2$. Which one of the following is an equation of the resulting curve?

A $y = -4 - (x - 5)^2$

B $y = -4 + (x + 5)^2$

C $y = 4 - (x + 5)^2$

D $y = 4 + (x - 5)^2$

E $y = -6 - (x - 5)^2$

F $y = -6 + (x + 5)^2$

G $y = 6 - (x + 5)^2$

H $y = 6 + (x - 5)^2$

19. Curve C has equation $y = 30 - x^2$. Line L has equation $y = 5$. What is the area enclosed between C and L?

A $\frac{440}{3}$

B $\frac{470}{3}$

C $\frac{500}{3}$

D $\frac{530}{3}$

E $\frac{560}{3}$

20. How many solutions of the equation $3 \sin 3\theta = \sin \theta$ lie in the interval $-\frac{\pi}{2} \leq \theta \leq \pi$?

A 2

B 3

C 4

D 5

E 6

F 7

21. The line $y = x + k$, where k is a constant, is a tangent to the curve $y = 3x^2 - 5x + 6$. What is the value of k?

A -3

B -2

C -1

D 1

E 2

F 3

G 4

H 5

22. In a particular arithmetic progression: the 15th term is four times the 1st term; the 10th term is 11 less than twice the 5th term. What is the 3rd term of the progression?

A 8

B 14

C 17

D 20

E 23

F 26

23. Evaluate $\log_2\left(\frac{3}{2}\right) + \log_2\left(\frac{4}{3}\right) + \log_2\left(\frac{5}{4}\right) + \cdots + \log_2\left(\frac{128}{127}\right)$.

A -6

B 3

C 4

D 5

E 6

F $\log_2 127$

24. Circle C has equation $(x+1)^2 + (y-3)^2 = 2$. The length of the tangent from the circle C to the point P is $4\sqrt{2}$. What is the shortest distance from P to C?

A $4\sqrt{3}$

B $4\sqrt{3} + \sqrt{2}$

C $4\sqrt{2}$

D $\sqrt{2}$

E 10

25. What is the coefficient of x^5 in the expansion of $(1-2x)^7(1+2x)^7$?

A -2240

B -336

C -28

D 0

E 28

F 336

G 2240

26. Given that $\int_0^4 (3x+m)\,dx = 28$, what is the value of m?

A -2

B -1

C 0

D 1

E 2

F 3

G 4

27. The dimensions of a solid cuboid, in cm, are x, $4x$ and y. The volume of the cuboid is 800 cm^3. At this volume, the surface area of the cuboid has its optimum value. What is the area, in cm^2, of the face that has the largest area?

A $2(400)^{2/3}$

B 40

C 100

D 160

E 400

F $5(320)^{2/3}$